

Finite Size Gap Effects on the Modeling of Thermal Contact Conductance at Polymer-Mold Wall Interface in Injection Molding

L. SRIDHAR, K. A. NARH

New Jersey Institute of Technology, Department of Mechanical Engineering, University Heights, Newark, NJ 07102

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ABSTRACT: Heat transfer in polymer processing by injection molding is affected by the thermal contact conductance at the interface between the polymer and the metal mold. The modeling of thermal contact conductance at such interfaces is simplified by the assumption of an isothermal condition at the two contacting surfaces. In this study we examine the validity of such an assumption for the case of an interface involving plastic (a low thermal conductivity material) and metal (a high thermal conductivity material). The study shows that at such an interface between materials of widely varying thermal conductivity, the conditions at the interface depart from the isothermal assumption, with the heat flux becoming more uniform and the temperature difference varying by a larger magnitude across the contact plane. This effect is more pronounced as the width of the gaps increases for the same area of contact. This suggests that the modeling of the contact conductance should be based on average temperatures for the contacting surfaces. © 2000 John Wiley & Sons, Inc. *J Appl Polym Sci* 75: 1776–1782, 2000

Key words: finite size gap effects; thermal contact conductance; injection molding; polymer–mold wall interface

INTRODUCTION

Imperfect contact at the interface between two surfaces introduces an additional conductance term, known as “thermal contact conductance,” in the computation of the flow of heat from one body to another. The imperfect contact, which is due to surface profile effects, gives rise to regions of gap interspersed with regions of contact at the interface. Since the regions of contact and gap are randomly distributed on any real surface, a parameter representing their effect, on the average, is required for engineering computations. The joint thermal contact conductance represents such a parameter, and has been widely studied^{1,2}

for metal–metal contacts. It should be noted that the term “contact conductance” is generally used in the literature for the joint thermal conductance; we follow the same usage for clarity, and the conductance at contacts is brought out specifically where applicable.

Yovanovich³ reviewed the thermal conductance models for contacts, gaps, and their combined effect at the joint. Conductance relations for contacts based on both elastic and plastic deformation were reviewed along with the gap conductance models. The joint conductance is then obtained as a combination of the conductance of contact and gap, in parallel. Joint thermal conductance models developed in this manner have been verified experimentally.⁴ The results of joint thermal conductance measurements at metal–metal interfaces show that the joint thermal conductance has a functional relationship with the

Correspondence to: K. A. Narh (narh@admin.njit.edu).

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contact pressure that follows a trend similar to that between the conductance due to contact, and the contact pressure. Development of joint thermal conductance models that consider the conductance due to contact, and gap, to act in parallel assume that the contacting surfaces are isothermal. The experimental verification of models based on this assumption shows that the resulting error is within acceptable limits. The experimental validation of these models has been mainly for metal–metal interfaces where the thermal conductivity of the contacting surfaces are relatively large and the surface effects are mainly due to roughness.

Improvements in simulation software and more accurate modeling of heat transfer in design applications, such as electronic cooling, have led to the study of thermal contact conductance (also referred to by its inverse, the thermal contact resistance) at interfaces between metals and polymers/elastomers.^{5–12} Such interfaces are typically between materials of widely varying thermal conductivity. From the perspective of processing and product design applications, metals that are most likely to be used (steels, aluminum) have thermal conductivity in the range of 35–250 W/m-K.¹³ Plastics, on the other hand, have thermal conductivity generally in the range of 0.1–0.4 W/m-K,¹⁴ while thermal conductivity of elastomers are in the range 0.5–1.5 W/m-K. Seyed Yagoobi et al.¹⁵ studied the thermal contact conductance at a paper–metal interface and showed that the contact conductance was a function of the logarithm of the contact pressure. This result is at variance with the results of thermal contact conductance models for metal–metal interfaces where the contact conductance is a function of the contact pressure raised to a power n , with n in the range 0.9–0.97. Marotta and Fletcher⁵ showed that the contact conductance at selected polymer–aluminum interfaces also did not follow the trend given by either the elastic or the plastic models for contact conductance. Furthermore, the results of Rhee et al.,⁸ and Narh and Sridhar⁹ show that the contact conductance at conforming plastic–metal surfaces is of the order of $0.125\text{--}0.25 \times 10^5$ W/m²-K, which is more than an order of magnitude lower than that reported by Marotta and Fletcher,⁵ for surfaces that were not specially prepared to be conforming. At such interfaces, the effect of the surface waviness is expected to be a dominant factor in determining the magnitude of contact conductance,¹⁰ resulting in a departure from the established contact conductance models.

One of the effects of surface waviness is to increase the width of finite size gaps. The interface, in such cases, can be assumed to consist of regions of relatively good contact (with gap dimensions of the order of the surface roughness) and regions of finite size gaps. The thermal conductance at a joint is then a combination of the conductance of the two regions. In order to improve the heat transfer model for simulation and thermal analysis, it is necessary to be able develop a joint thermal conductance model that better approximates the phenomenon. In this study, the effect of the dimensions of the gap at the interface on the steady state heat transfer is studied numerically to determine if the assumption of an isothermal contacting surface can be used to combine the conductance of the two regions with widely varying thermal conductivity—to develop a joint thermal contact conductance model.

CONTACT CONDUCTANCE BEHAVIOR IN INJECTION MOLDING

Contact conductance at the polymer–mold wall interface is a parameter to be considered in analysis of injection molding processes. Unlike most cases of steady state contact conductance phenomena, in the case of injection molding, Yu et al.⁶ reported results that showed that the contact conductance varied widely in magnitude from the start of filling (high value) to the time of ejection (low value). Kamal et al. also noted a variation in the contact conductance (termed heat transfer coefficient by them) but limited their study to the filling period. They obtained an average contact conductance value as a function of the melt velocity. The observed variation in the contact conductance has been recently linked to the shrinkage in the thickness direction by Sridhar et al.,¹⁶ who used simulation results to predict the behavior of the gap at the interface between the vitrifying polymer and the mold wall, in the case of injection molding. The results show that the interface consist of regions of finite sized gaps formed by the thickness-direction shrinkage and regions of good contact formed during the initial filling phase. The area of these regions at any instant of time in each molding cycle depends on the rate of heat transfer, unbalanced cooling effects, and nonuniform shrinkage. Using ultrasound techniques, Wang et al.¹⁷ confirmed the existence of interfacial gap due to the thickness direction shrinkage though the size of the gaps was not quantified.

These results together suggest that the contact conductance is a function of the thickness direction gap between the plastic and the mold. The magnitude of this gap varies from a small value at the beginning of filling and reaches a maximum at the end of the postfilling stage (i.e., at ejection). Furthermore, due to effects such as unbalanced cooling, nonuniform shrinkage, adhesion between the mold and polymer, body forces, etc., the magnitude of the gap need not be the same at every point on the plastic surface. The shrinkage induced gap builds up in magnitude that is much larger than the gaps due to mold wall surface roughness. The surface can thus be modeled as consisting of areas where the plastic touches the metal (small gap dimensions; the contact conductance being governed by the mold wall roughness) and areas of finite-size gaps. The dimensions of the finite size gaps (width and thickness) vary with the time instant in the injection molding cycle.

In the present work, the effect of the joint thermal conductance on the heat transfer at the interface has been studied for different magnitudes of gap dimension. A steady state analysis has been performed with the limited objective of studying the effect of the contact conductance for specified gap dimensions, using values of thermal conductivities of the contacting surfaces that are typical for injection molding of thermoplastics.

STEADY STATE PROBLEM FORMULATION

Assuming a rectangular geometry for the gaps, the study was conducted for a two-dimensional model with the plastic and the mold being modeled by two contacting isotropic regions, as illustrated in Figure 1. Region 1 represents the metal region, with higher thermal conductivity k_1 , while region 2 represents the plastic region, with lower thermal conductivity k_2 . The thermal conductivity of both regions are assumed to be independent of temperature. The governing equations and boundary conditions for the steady state problem can then be written as:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0; \quad -a < y < a; \quad 0 < x < b \quad (1)$$

$$T(x, -a) = T_1; \quad T(x, a) = T_2 \quad (2)$$

$$-k_{1,2} \left. \frac{\partial T}{\partial x} \right|_{x=0,b} = 0 \quad (3)$$

$$\begin{aligned} -k_1 \left. \frac{\partial T}{\partial y} \right|_{y=0+} &= H_c [T(x, 0+)] \\ &= -k_2 \left. \frac{\partial T}{\partial y} \right|_{y=0-}; \quad x \in C \quad (4) \end{aligned}$$

$$\begin{aligned} -k_1 \left. \frac{\partial T}{\partial y} \right|_{y=0+} &= H_g [T(x, 0+) - T(x, 0-)] \\ &= -k_2 \left. \frac{\partial T}{\partial y} \right|_{y=0-}; \quad x \in G \quad (5) \end{aligned}$$

where T is temperature ($^{\circ}\text{C}$), x and y are two coordinate directions, a and b are the length and width of each region, H_c is the conductance for the regions of good contact, and H_g the conductance for the gap at the interface, respectively. Equations (5) and (6) thus define the modeling of the interface that was modeled as a very thin layer divided into regions of contact C and regions of gaps G . A fixed value of $10^5 \text{ W/m}^2\text{-K}$ was assumed for H_c , based on the results of Narh and Sridhar,⁹ who found that the variation of thermal contact resistance (the inverse of contact conductance), with pressure at surfaces prepared such that the metal roughness governs the contact resistance, is of a much smaller magnitude compared to the variation in shrinkage controlled gap thermal conductance. Typically, interface gaps in plastic processing are in the range of 1–100 μm with the surface roughness of mold being less than one micron. The value of H_g used was in the range 10^3 – $10^4 \text{ W/m}^2\text{-K}$ for air-filled gaps of 3, 15, 30, and 60 μm , based on the following equation:

$$H_g = k_g / (\lambda + g_1 + g_2). \quad (6)$$

where λ is the gap thickness and g_1 and g_2 are temperature jump distances, their magnitudes depending on the interstitial medium.

A finite element mesh, illustrated in Figure 1, was created using 8-noded quadrilateral elements with the mesh size chosen such that the effect of the variation at the interface can be captured in the numerical results. The problem was solved using the finite element software ANSYS.

RESULTS AND DISCUSSION

The model was solved for typical values of k_1 and k_2 with $k_1 = 30 \text{ W/m-K}$ for steel and $k_2 = 0.2$ for

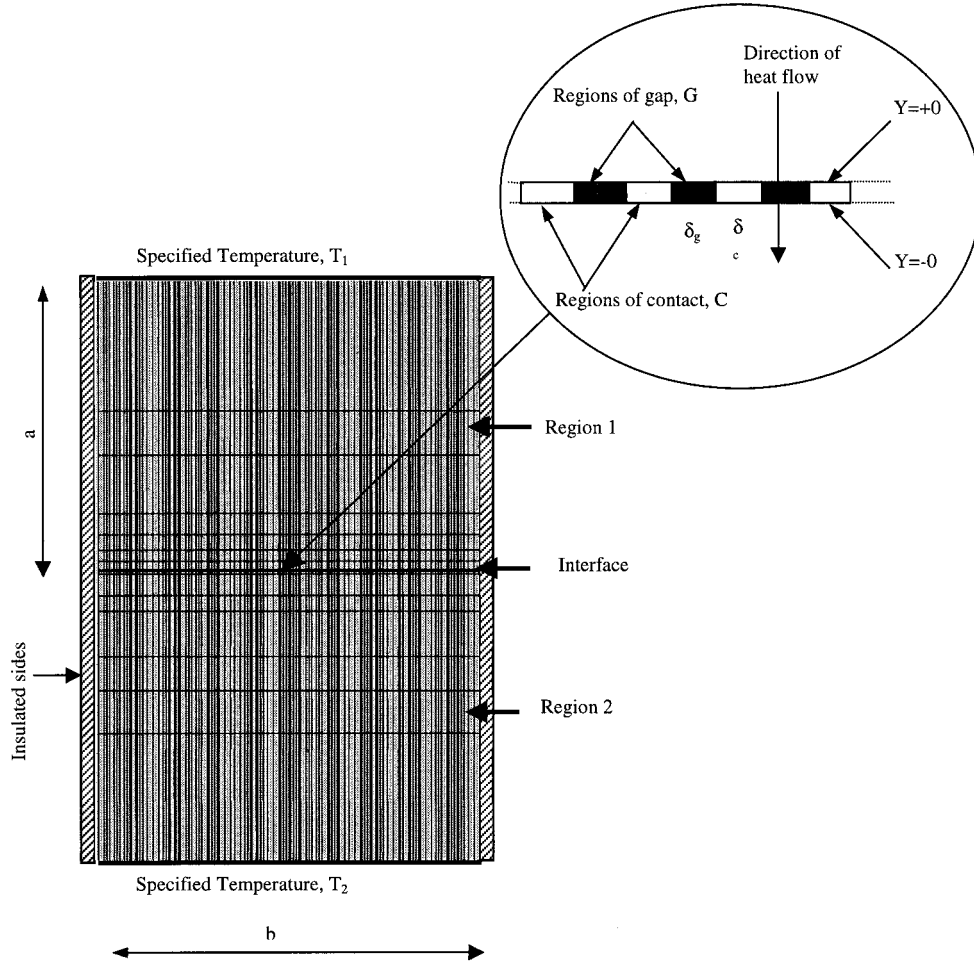


Figure 1 Finite element mesh of the two-dimensional geometry analyzed, showing the interface and the boundary conditions. Note that the thickness of the interface has been exaggerated to show that the interface is divided into regions of good contact and gap depending on the conductance value used.

polymers. The solution was obtained for different values of the gap and contacting area widths (δ_g , δ_c) and for each value of gap thickness (H_g). The widths and gap dimensions are based on predicted deformation characteristics in injection molding.¹⁶ The smallest gap dimensions representative of conditions obtained at the end of filling. The analysis results were obtained in the form of average heat flux in the y direction at the interface and the temperature difference across the interface. The results were normalized with respect to the heat flux and temperature differences obtained with the assumption of isothermal contacting surface. For the steady state case studied, the isothermal condition assumption leads to the following equations, which define the heat transfer across the interface:

$$Q = \frac{(T_1 - T_2)}{\left(\sum_C \delta_c H_c + \sum_G \delta_g H_g \right)^{-1} + \frac{a}{b} \left(\frac{1}{k_1} + \frac{1}{k_2} \right)} \quad (7)$$

$$\Delta T = (T_1 - T_2) - Q \frac{a}{b} \left(\frac{1}{k_1} + \frac{1}{k_2} \right) \quad (8)$$

$$q_c = H_c \Delta T; \quad q_g = H_g \Delta T \quad (9)$$

where Q is the total heat flow from region 1 to region 2, ΔT is the isothermal temperature difference across the interface, and q_c and q_g are the corresponding heat fluxes.

Figures 2 and 3 show the numerical results plotted in the form of the ratios of heat flux across

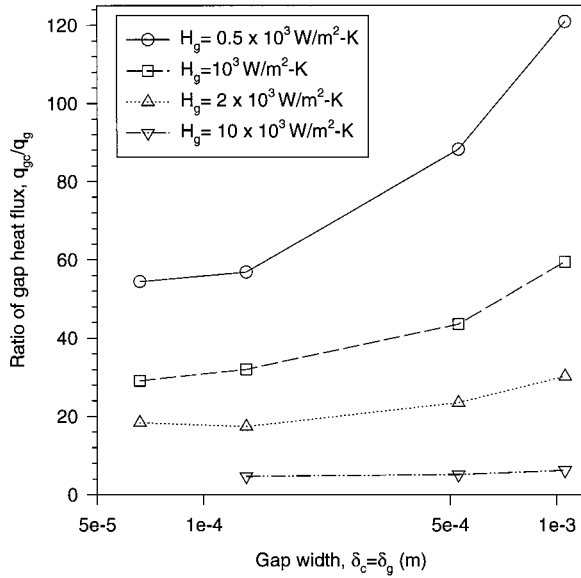


Figure 2 Plot of the ratio of the gap heat flux for different values of the gap width δ . Values of q_{gc}/q_g close to unity indicate near isothermal conditions.

the gap, and that across the contact regions, to the heat flux values computed from eqs. (7)–(9). Based on the numerical analysis, q_{cc} and q_{gc} represent the average heat flux in the y direction, and ΔT_c and ΔT_g represent the average temperature difference across the contact and gap regions, respectively. The q_c and q_g are given by eq. (9). The ratios are plotted against the gap width δ_g for

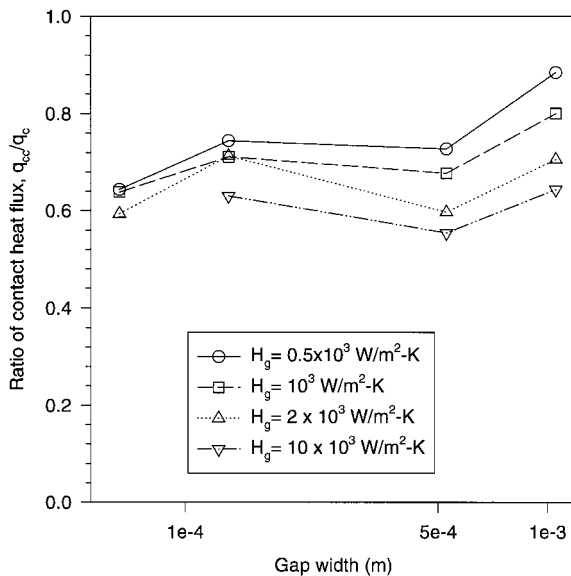


Figure 3 Plot of the ratio of the contact area heat flux for different values of the gap width δ .

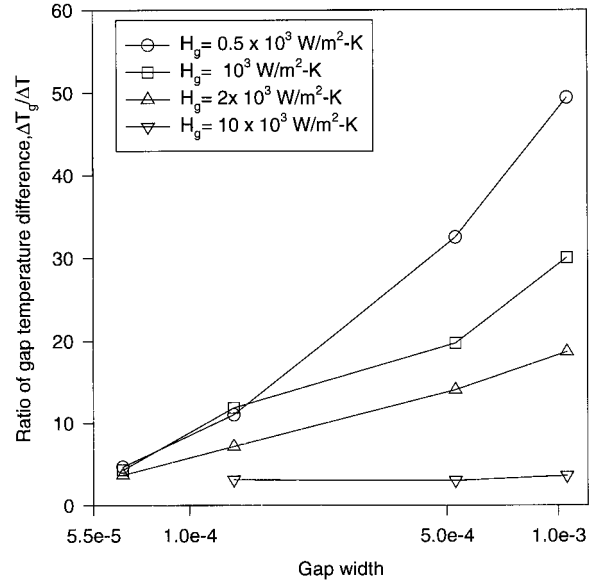


Figure 4 Plot of the ratio of the gap area interfacial temperature difference for different values of the gap width δ . Values of $\Delta T_g/\Delta T$ close to unity indicate near isothermal conditions.

different values of gap thickness in terms of H_g ; the larger the value of H_g the smaller the gap thickness. For the case of $k_1 = 30$, Figure 2 shows that as the gap width increases, the gap heat flux increases with respect to the flux computed under isothermal conditions. This increase is more pronounced the smaller the value of H_g (i.e., the larger the gap thickness). On the other hand, Figure 3 shows that the contact heat flux displays a much smaller variation with respect to the flux, under isothermal conditions, as the gap width increases. As the gaps become wider, the heat flux across the interface becomes more uniform.

Figures 4 and 5 show the dimensionless interfacial temperature drop across the gap and contacting areas, respectively, as a function of gap width. ΔT is given by eq. (8) while ΔT_g and ΔT_c were obtained from the numerical analysis. Figure 4 shows that the numerical gap temperature difference increases with respect to the isothermal condition ($\Delta T_g/\Delta T \cong 1$) temperature difference, as the gap width increases. As in the case of the heat flux, the increase is more pronounced for lower values of H_g , corresponding to higher gap thickness. Figure 5 shows that as the gap width increases, the ratio of the numerical contact temperature difference varies only marginally with respect to the isothermal condition temperature difference, with no particular trend.

Thus, as the width and thickness of the gaps are increased, the heat transfer across the interface departs from the isothermal condition assumption. This effect becomes significant when the gap thickness is approximately 15μ ($H_g = 10^4 \text{ W/m}^2\text{-K}$). At larger gap thickness, increasing the gap width will cause further deviation from the isothermal condition. Regarding the heat flux and temperature difference at the regions of contact, one would also expect a deviation from the isothermal condition as the gap dimensions are increased. However, the numerical results for the heat flux seem to indicate a small trend *toward* the isothermal condition. This can be explained in terms of the elements used in the numerical analysis: As the gap dimensions were decreased, the number of elements available in each contact/gap region also decreased, introducing a slightly larger error in the numerical estimations. This effect is more pronounced in the contact regions due to the smaller interfacial temperature difference at these locations (see Fig. 6).

Figure 6 shows the temperature distribution on either side of the interface normalized with respect to the total temperature difference ($T_1 - T_2$), for $\delta_g = 67 \times 10^{-6} \text{ m}$. A similar profile is obtained for all the other gap dimensions analyzed. The figure shows that the variation in the dimensionless temperature on the side with high thermal conductivity (region 1) is much smaller compared with that in the region with smaller

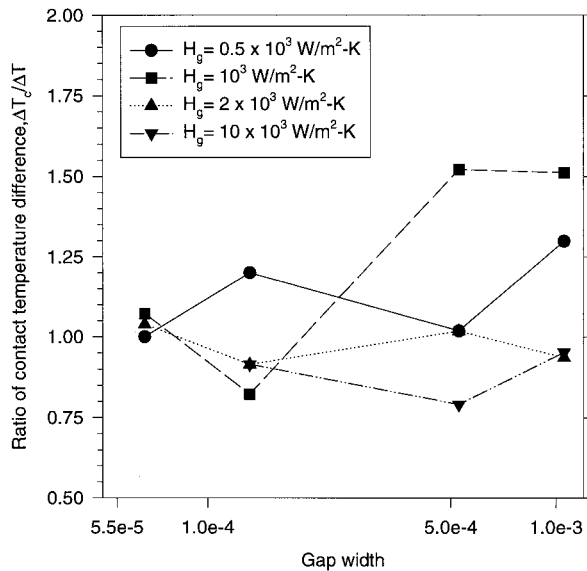


Figure 5 Plot of the ratio of the contact area interfacial temperature difference for different values of gap width δ .

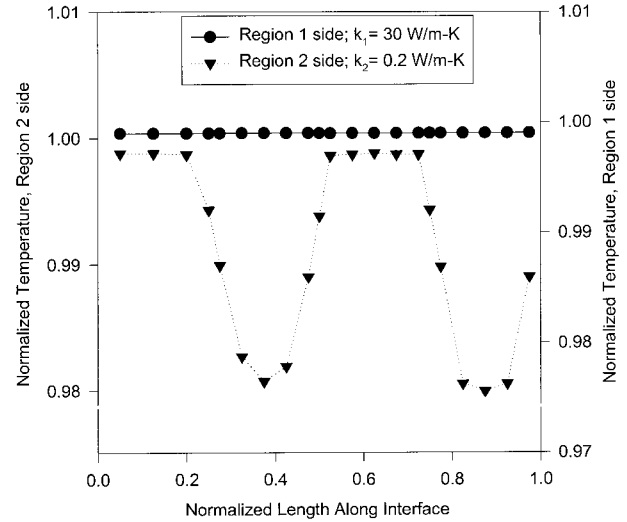


Figure 6 The variation of the interface surface temperature for the two regions for the case of $\delta = 67 \times 10^{-6} \text{ m}$. The temperatures are normalized with respect to the overall temperature difference ($T_1 - T_2$) and the length scale is normalized with respect to the length of two contiguous areas of gap and contact.

thermal conductivity (region 2). Furthermore, the difference in temperature variation between the two sides of the interface increases as the gap width and thickness increase.

This result may explain the small value of contact conductance reported by Yu et al.,⁶ and also by Kamal et al.,⁷ for the later stages of the injection molding cycle. For the case of isothermal surface assumption of Yovanovich,³ the higher magnitude of the conductance at the regions of contact H_c will dominate the value of combined conductance at the interface for equal areas of good contact and gap, as can be deduced from the denominator of eq. (7). As has been reported by Rhee et al.⁸ and Narh and Sridhar,⁹ from their experimental results, the contact conductance at the regions of good contact in case of the interface between PS and metal is very high, and hence would dominate the value of the combined contact conductance. However, the values reported by the Yu et al are more than an order of magnitude lower.

CONCLUSION

The numerical analysis shows that as the width and thickness of the gap at the interface between a low thermal conductivity material (typical of thermoplastics used in injection molding) and a

high thermal conductivity material (typical of the mold material) increase the conditions at the interface depart from the isothermal condition. Thus, the assumption of isothermal condition at the interface, often used to combine the conductance due to contacts and finite size gaps using a parallel circuit analogy, may result in errors in a model for the joint thermal conductance for plastic-metal interfaces. The departure from isothermal conditions increases as the width of the gaps increases, even though the total areas of contacts and gaps remain constant. It should be noted that the areas of contacts referred to here are actually regions with microscopic gaps as opposed to the finite size gap region. The results suggest that the joint thermal conductance should be computed based on average temperatures for the two contacting surfaces. The results also show that as the gap width and thickness increase, the interfacial heat flux tends to become more uniform while the interfacial temperature drop varies widely. In certain cases the temperatures near the interface may approach the transition value. In that case, the material near the regions of contact may be at or above the transition temperature while that near the region of gap may be below the transition temperature. For the case where the heat is flowing in a direction opposite to that considered in this analysis, such as in the injection molding process, the reverse will also be true; the plastic material at the gap region may be at a higher temperature than that predicted under the isothermal assumption. This conclusion has serious implications for the prediction of the surface temperature of the plastic part, one of the parameters required for the prediction of cooling time in injection molding simulation.

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